APPLICATION OF THE METHOD OF CHARACTERISTICS FOR SOLUTION OF THE UNSTEADY NON-ISOTHERMAL NON-ADIABATIC FLOW EQUATIONS

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ABSTRACT

Unsteady flow of fluids in pipelines can adequately be represented by a system of partial differential equations, of first- or second-order depending on the type of problem. Such systems may either be parabolic or hyperbolic. Many numerical methods are available and have been used for the solution of these equations. A new approach based on the Gamma Delta method developed by Flatt, was used. The three basic partial differential equations of flow were derived for unsteady quasi-one-dimensional flow of real gases through a non-rigid non-constant cross-section area pipe. The QUANT software for thermodynamic and transport properties of real gases was used. The software is based on the virial equation of state and also contains the coefficients required for the Gamma Delta method. A flow dependent explicit equation was used to calculate the friction losses. Numerical solution of the basic equations was effected using the method of characteristics. A computer coding using the C programming language was developed, for modelling of unsteady and transient flow following linebreak in high-pressure natural gas pipelines.

INTRODUCTION

This study was aimed at developing a computer model for analysis of transient and unsteady flow following linebreak in high-pressure natural gas pipelines. This type of flow requires a non-isothermal non-adiabatic treatment because of the big temperature drops involved. There are many computer codes available for analysis of unsteady flow of fluids in pipelines. However, only a few are known to be applicable to linebreak situations and their scope is limited. Discrepancies between different models which

have been developed have mainly centred on the assumptions made in developing the basic partial differential equations of flow, and subsequent simplifications; the thermophysical model used; representation of various terms in the equations such as the friction term; and the numerical method used for solution of the basic equation.

The process of developing a computer model for transient analysis of gas pipelines entails three main steps, namely formulation of the basic governing equations, solution of the equations and finally validation of the model with experimental data. Unsteady flow of compressible fluids in pipelines is described by a set of three partial differential equations, derived from the principles of conservation of mass, conservation of momentum and conservation of energy. The fluid properties are described by an equation of state. These together with appropriate auxiliary conditions, determine the mathematical state of the fluid. Many assumptions and simplifications are involved in the process of formulating and manipulating these equations. It is generally preferred to keep the equations as simple as possible, without significantly reducing the accuracy of results in a particular model, in order to economise on computational labour and time and also to minimise the computer memory requirement. Most cases of unsteady one-dimensional flow, where disturbance propagation velocities do not vary significantly, are characterised by quasi-linear hyperbolic partial differential equations for continuity, momentum and energy.

Four important assumptions and simplifications with regard to flow dimension, flow phase, Fluid Structure Interaction (FSI), and minor losses including the limitations arising from them were discussed [1]. In one-dimensional flow, the components of the fluid velocity in the circumferential and radial directions are ignored. Multi-phase flow in general is a very complicated phenomenon. A gas-liquid mixture may be treated as a pseudo-fluid, if the mixture and its motion may be treated as homogeneous. FSI and effects arising from minor losses and changes in cross section were also discussed [1]. No further simplification was made on the basic equations.

A new approach was used, in which the three basic partial differential equations of flow were derived using the Gamma Delta method, developed by Flatt [2]. This method is briefly described in the section which follows

and full derivation of the equations was made [1]. The QUANT software for thermodynamic and transport properties of real gases was used. The method of characteristics was used for solution of the basic equations of flow.

THE BASIC EQUATIONS OF FLOW

The basic equations for unsteady quasi-one-dimensional flow of real gases through non-rigid and variable cross-sectional area pipes are considered, using the Gamma Delta method, developed by Flatt [2]. These equations are valid for three fluid forms: gas (perfect and real), homogeneous liquid/ vapour mixture and liquid. These equations simplify considerably problems where several of the three fluid forms appear simultaneously Formulation of the energy equation for unsteady flow of fluid in pipes has commonly contained either specific internal energy or specific enthalpy. Each of the above mentioned dependent variables is related to the other dependent variables e.g. pressure (p), density (ρ), and temperature (T) by a caloric equation of state which is often a complicated non-linear empirical correlation in integral form. This procedure sometimes involves as many as 20 or more fluid dependent coefficients. With the help of the two non-dimensional coefficients γ and δ, the specific internal energy and specific enthalpy have been eliminated from the energy equation, resulting in considerable computing economy. The three basic equations of conservation for unsteady flow were derived from first principles [1], assuming that the cross-sectional area of the pipe varies with time (t) as well as with axial position (x). The resulting continuity equation is as follows:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = \xi \qquad (1)$$

where ξ is the term representing the variation in cross-sectional area of the pipe due to its geometry and elasticity of the pipe material. In equation form, ξ is defined as follows:

$$\xi = -\rho\left[\frac{1}{A}\frac{\partial A}{\partial t} + \frac{u}{A}\frac{\partial A}{\partial x}\right] \qquad (2)$$

For rigid and cylindrical pipes $\xi = 0$. The momentum equation was derived, obtaining the following equation:

$$\frac{\partial u}{\partial t}+\frac{1}{\rho}\frac{\partial p}{\partial x}+u\frac{\partial u}{\partial x}=-\frac{\omega}{\rho A\cos\psi}-g\sin\theta \qquad (3)$$

For a uniform diameter pipe, Cosψ = 1. The resulting energy equation is as follows:

$$\frac{\partial p}{\partial t}+\frac{u\partial p}{\partial x}-a^2\left(\frac{\partial \rho}{\partial t}+\frac{u\partial \rho}{\partial x}\right)=(\delta_s-1)\frac{1}{A}\left(\Omega+\frac{\omega u}{\cos\Psi}\right) \qquad (4)$$

The continuity equation (1) contains the partial derivative ofρ with respect to t, while the momentum equation (3) contains the partial derivative of u with respect to t. For convenience of numerical solution of the three equations, the energy equation was rewritten such that the term containing partial derivative with respect to t, was that of p only. This condition was achieved by substituting the continuity equation (1) into equation (4). The resulting equation, which was used in the numerical solution is as follows:

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+a^2\rho\frac{\partial u}{\partial x}=(\delta_s-1)\frac{1}{A}\left(\Omega+\frac{\omega u}{\cos\psi}\right)+a^2\xi \qquad (5)$$

Equation (5) is simpler and more convenient than equation (4). The former equation was therefore used in the solution of the three simultaneous equations of conservation. However, in deriving the characteristic and compatibility equations for the numerical method of characteristics, equation (4) was used instead of equation (5) because the latter equation failed to produce a unique solution for the gradient of the characteristic equations.

The three basic equations of conservation [equations (1), (3) & (5)] were written in such a way that the solution of pressure, velocity density and temperature could be obtained. This was done by using the equations of state and some other thermodynamic relationships the model used to represent the transient event. Over five dozen equations of state are known to exist, which represent the liquid, vapour and liquid-vapour regions. The two general approaches used to develop the equation of state: the theoretical approach gives a higher accuracy; while the empirical approach does not. The criteria for selection of equation of state for a particular flow situation were also discussed [1].

QUANT, a commercial computer programme for thermodynamic and transport properties of real gases and their mixtures was used. The programme is based on the virial equation of state and contains coeficients required for the Gamma Delta method. The programme delivers properties of elements, compounds and any of their mixtures; but is restricted to the gaseous phase so far.

FRICTIONAL FORCE

Modelling of frictional effects may be done either by numerically evaluating Darcy's frictional factor, f_D, in equation (6) using known flow field data or by using experimental correlations. The friction term in the basic equations is denoted by ω and is defined as the friction force per unit length of the pipe opposing the flow. After assuming that the minor losses are negligible, the frictional force per unit length is expressed by the empirical relationship;

$$\omega = \frac{A}{d}\rho f_D \frac{u|u|}{2} \qquad (6)$$

At least one and a half dozen friction factor expressions have been identified [1]. For laminar flow (Re<2100), the Hagen-Poiseiulle equation is used. For fully developed turbulence, the rough pipe law which assumes that the friction factor is solely dependent on the pipe roughness and size is used. For partially developed turbulence either the smooth pipe law or the Blasius form of the smooth pipe law are used. Here the friction factor is assumed to be only dependent on the fluid properties and pipe size. For the transition zone between partially and fully developed turbulence a combination of both the rough and smooth pipe laws is used. The Colebrooke equation has been universally adopted for this regime. However, there are numerous other equations which could be solved explicitly, and with almost the same accuracy as the Colebrooke equation. The key factor in applying a flow-dependent friction factor is the determination of which flow regime is to represent the flow at a particular point and time.This situation is exacerbated by the fact that, in many practical flow situation many flow regimes exist and thus different equations have to be used. At least two expressions for friction factor, including that by Chen [3], are known which cover the whole range of Reynolds numbers and pipe roughness and which produce results which are nearly the same as those produced by the Colebrooke equation. The Chen equation was used. The friction factor was calculated for each grid point as a function of Reynolds number A frequency-dependent

friction factor was not used because there has not yet been any satisfactory results to justify its application. A two-phase friction factor was not used in this model. Since it was decided earlier not to include the effects of FSI in this model, its effects on friction factor were also neglected. A second-order approximation was used to calculate the friction force.

HEAT TRANSFER

The heat transfer term, Ω, is defined as the heat flow into, or from the pipe wall per unit length of pipe. Although this term is considerably smaller in magnitude than the friction term, its effect is considerable especially when considering long distance pipelines. In isothermal flow the energy equation becomes redundant except during the calculation of the value of the heat transfer, Ω. In adiabatic flow, it is assumed that there is no net flow of heat through the pipe, even though some heat transfer will take place between the fluid and its surroundings. Isothermal flow relates to slow dynamic changes. When a pipe is long and the change is relatively gradual, the fluid will tend to come to thermal equilibrium with the pipe. Adiabatic flow relates to fast dynamic changes in the fluid, where it is assumed that the pressure changes occur instantaneously allowing no time for heat transfer to take place between the pipe and the surroundings. For accuracy a non-isothermal non-adiabatic heat transfer model was used.

Two different approaches to calculation of heat transfer for the gas were considered. In the first approach, one of the relationships between the dimensionless numbers, namely Reynolds number (Re), Prandtl number (Pr), Nusset number (Nu) and Stanton number (St) was used to calculate the convective heat transfer coefficient across the boundary layer. In the second approach the adiabatic wall temperature and recovery factor were used to calculate the heat transfer. This method was derived from first principles [1]. The difference between the two approaches is the way in which the heat transfer between the fluid and the inner wall of the pipe are calculated. For the rest of the system, the analysis is the same for both approaches.

NUMERICAL METHODS OF SOLUTION

The most commonly used numerical methods for fluid flow analysis are the methods of characteristics, finite-difference, finite-element,

flux-difference splitting schemes, the method of lines and the wave-plan method. The basis of finite-difference formulations is the differentials of the dependent variables appearing in the partial differential equations, expressed in approximate expressions so that a digital computer which performs only standard arithmetic and logical operations can be employed to obtain a solution. The finite-difference approximations are used to replace the derivatives that appear in the partial differential equations. Finite-difference methods are therefore categorised in the two types, namely, explicit and implicit formulations. Solution of explicit equations is simpler than the implicit equations. However, implicit formulations are more stable than explicit formulations. The method of characteristics, flux-difference splitting schemes, method of lines and wave-plan method are based on finite-difference formulation. Many different finite-difference methods, ranging from single-step first-order accurate to four-step fourth-order accurate schemes, have been developed for the fluid transient equations.

The method of characteristics is commonly used as a numerical method for quasi-linear hyperbolic systems in two independent variables. By an appropriate choice of coordinates, paths can be defined in the x-t plane, called characteristic lines, along which the system of partial differential equations is converted into a system of ordinary differential equations that may be solved by standard single step finite-difference methods for ordinary differential equations. The basic rationale underlying the use of characteristics is that by an appropriate choice of coordinates, the original system of hyperbolic equations can be replaced by a system whose coordinates are the characteristics (natural method of characteristics). The use of this method becomes particularly simple when applied to two equations in two dependent variables.

One of the major drawbacks of the method of characteristics appears when the dependent variables are required at fixed time intervals and a two-dimensional interpolation in the characteristic net is required. This drawback has been overcome by the hybrid or mesh method of characteristics, also called the method of specified time intervals, which solves the characteristic equations on values for the dependent variables at specified time-distance coordinates. With the mesh points defined in advance, and the interpolation taking place as computation advances, it becomes a one-dimensional interpolation. Although the method of

characteristics is most ideal for the solution of quasi-linear hyperbolic equations with two dependent variables on characteristics, which is the natural coordinate system, a great deal of effort has been made to extend the method to other more complicated cases. The above effort includes the extension for calculating three dependent variables encountered in transient non-isothermal gas flow. Finite-element methods have not been used widely for fluid transient analysis. The most recent finite-element method for fluid transients is the method of Bisgaard -Sørensen -Spangenberg [4].

The focus of this study was to model a linebreak in high-pressure gas pipeline. However, with appropriate specification of the boundary conditions the model should be able to analyse any other flow situation. This method was therefore chosen to accurately represents shock waves and accommodate the varying speeds of the waves without smearing the details or overshooting. The method of characteristics represents many pipe flow situations more accurately and it is the most popular method of solution for pipe flow problems.

THE BASIC EQUATIONS BY THE HYBRID METHOD

The theory of the method of characteristic has been described by among others Courant and Friedrichs [5], Lister [6] and Ames [7]. Two common hybrid methods are those by Courant, Isaacson and Rees [8] which treats first-order problems and Hartreee [9] which treats second-order systems. According to Ames [7] the Hartree method is more accurate than the Courant-Isaacson-Rees method. Ames [7] also stated that the former method could be applied to second-order systems with only minor changes. Referring to Fig.1, the hybrid method starts by assuming that the solution is known at the mesh points on time level t. The intersections of the characteristic lines with the time level t line i.e. points Q, R and S are unknown. These together with the values of the dependent variables at point P are determined using the characteristic and compatibility equations. Interpolation for the values of the dependent variables and the positions of points Q, R and S is necessary at each step.

The first step in the method of characteristics solution is to convert the basic partial differential equations of flow into ordinary differential equations. Two most common methods of achieving this are the matrix

transformation method, such as the one used by Tiley [10] and that of multiplying the basic equations by an unknown parameter and summing them. The latter method was used by Lister[6], Wylie and Streeter [11] for isothermal flow (only two equations) and by Zucrow and Hofman [12] for non-isothermal flow (three equations). The method used by Zucrow and Hoffman was adapted for this study because of its simplicity, mathematical rigour and also because the equations used in this study are very similar to those used by Zucrow and Hoffman [12]. The common practice in the method of characteristics solution is to use first-order and linear approximations to calculate values at the next time level. Values obtained in the first-order calculation are used as initial estimates for the iterative solution in the second-order approximation. In the case of hybrid methods, the first step is to find the positions of the intersections of the characteristic curves with the distance axis at time t, points Q, R, and S. This also could be done using either first- or second-order approximation.

For the first-order method, points Q, R and S were calculated using the characteristic equations and the values of u and a at point M initially, and the averages between the values at M and those estimates at P in subsequent iterations. The fluid properties at Q, R and S were calculated using linear interpolation between those at the respective surrounding grid points. A first-order approximation was used to calculate the properties at the next time step i.e. point P. Taylor's theorem was used to derive equations for quadratic interpolation so that new values of the fluid properties could be calculated at the bases of the characteristics. The results were used as variables in the characteristic and compatibility equations. Values calculated using the first-order approximation were used in the first iteration of the second-order approximation.

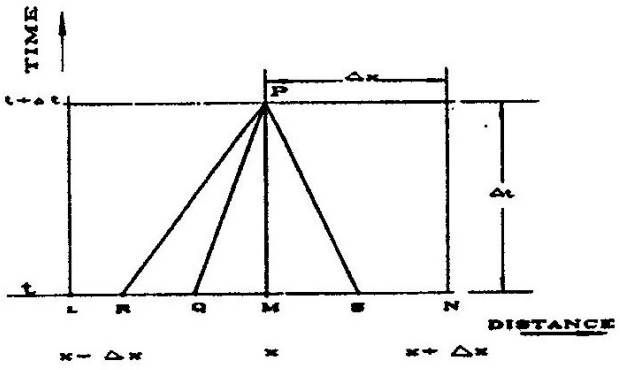


Fig. 1 Hybrid Method of Characteristics Solution Grid

The basic equations for unsteady flow equations (1), (3) and (5) were further simplified to the following equations:

Continuity equation

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0 \tag{7}$$

Momentum equation

$$\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} + u\frac{\partial u}{\partial x} = -\frac{\omega}{\rho A} - g\sin\theta \tag{8}$$

Energy equation

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + a^2\rho\frac{\partial u}{\partial x} = \frac{1}{A}(\delta_s - 1)_{(}\Omega + \omega u_{)} \tag{9}$$

The characteristic and compatibility equations corresponding to equations (7), (8) and (9) were derived by multiplying equations (7), (8) and (9) by unknown parameters δ_1, δ_2 and δ_3 respectively, summing the products and equating it to zero. For the sake of convenience of mathematical manipulation the energy equation used is that given by equation (4), but after making the same simplifications as those made to equation (5). The use of equation (5) lead to a coefficient matrix in which the elements of one row were all zeros. This resulted in a situation whereby there was no solution for the gradient of the characteristic equations, λ. The derivation of the characteristic and compatibility equations was covered fully[1].

$$\lambda_o = \left(\frac{dt}{dx}\right)_o = \frac{1}{u} \tag{10}$$

$$\lambda_+ = \left(\frac{dt}{dx}\right)_+ = \frac{1}{u+a} \tag{11}$$

$$\lambda_- = \left(\frac{dt}{dx}\right)_- = \frac{1}{u-a} \tag{12}$$

Equations (10), (11) and (12) are the three characteristic lines namely the path line characteristic C_o and the right- and left-running Mach lines C_+ and C. respectively.

$$\left[\frac{A}{(1-\delta_s)dt}\right]dp-\left[\frac{a^2A}{(1-\delta_s)dt}-\frac{\omega u}{2\rho}\right]d\rho+\omega du=-\Omega-\omega u \tag{13}$$

$$\left[\frac{A}{dt}\right]dp+\left[\frac{\omega}{2}\left(\frac{u}{\rho}(1-\delta_s)+\frac{a}{\rho}\right)\right]dp+\left[\frac{\rho aA}{dt}+\omega\left((1-\delta_s)+\frac{a}{u}\right)\right]du=$$
$$\left[-u(1-\delta_s)-\rho\frac{a}{\rho}\right]\omega-\rho aAg\sin\theta-(1-\delta_s)\Omega \tag{14}$$

$$\left[\frac{A}{dt}\right]dp+\left[\frac{\omega}{2}\left(\frac{u}{\rho}(1-\delta_s)-\frac{a}{\rho}\right)\right]d\rho-\left[\frac{\rho aA}{dt}-\omega\left((1-\delta_s)-\frac{a}{u}\right)\right]du=$$
$$-\left[u(1-\delta_s)-\frac{\rho a}{\rho}\right]\omega+\rho aAg\sin\theta-(1-\delta_s)\Omega \tag{15}$$

Equations (13), (14) and (15) are the compatibility equations along the path line characteristic C_0 and the right- and left-running Mach lines C_+ and C_- respectively. According to the theory of characteristics [5], every solution of the original system of partial differential equations should satisfy the characteristic and compatibility equations. The converse is also true and therefore every solution of the characteristic and compatibility equations must satisfy the original system of partial differential equations.

Whereas the natural method of characteristics is unconditionally stable, the hybrid method of characteristics is only conditionally stable. The stability criterion used is that of Courant, Friedrichs and Levy which states that the domain of dependence of the exact solution is contained within the domain of dependence of the numerical solution. The Courant-Friedrichs-Levy stability criterion is represented as:

$$\frac{\Delta t}{\Delta x}\leq\frac{1}{(|u|+a)_{\max}} \tag{16}$$

The solution of characteristic and compatibility equations could be obtained using either a first- or second-order approximation such as the trapezoidal rule.

SOLUTION OF THE CHARACTERISTIC AND COMPATIBILITY EQUATIONS

(i) Determination of the positions Q, R and S by first-order approximation

The mathematical structure used to obtain the solution of the characteristics is given below:

$$x_Q = x - u_M^t \Delta t \tag{17}$$

$$x_R = x - \left(u_M^t + a_M^t\right)\Delta t \tag{18}$$

$$x_s = x - \left(u_M^t - a_M^t\right)\Delta t \tag{19}$$

The subscripts and superscripts denote the point on the x-t plane time level respectively (refer Fig. 1).

(ii) Determination of the fluid properties at Q, R and S by linear interpolation

It is customary to assume that the characteristic lines are positioned as shown in Fig. 1. However, this may not necessarily be the same always. The only properties which were approximated using this method are ρ, p and u. The remaining fluid properties were calculated by the QUANT software using the values of ρ and p obtained in the first-order approximation as input values. The equations for first-order approximation of a fluid property, say p at point Q was as follows:

If $x_Q < x$

$$p_Q^t = \left(\frac{x - x_Q}{\Delta x}\right) p_{x-\Delta x}^t + \left[1 - \left(\frac{x - x_Q}{\Delta x}\right)\right] p_x^t \tag{20}$$

If $x_Q > x$

$$p_Q^t = \left(\frac{x_Q - x}{\Delta x}\right) p_{x-\Delta x}^t + \left[1 - \left(\frac{x_Q - x}{\Delta x}\right)\right] p_x^t \tag{21}$$

Similarly, the other fluid properties and the properties at the other positions were calculated by replacing the p and Q in equations (20) and (21) by the

other fluid properties and positions respectively.

(iii) Calculation of ρ, u and p at position P using first-order approximation

The values of ρ, u and p at position P are obtained by expressing the three differential equations (13), (14) and (15) in finite differences and solving them simultaneously. The fluid properties used to calculate the coefficients of the equations are evaluated at position M.

(iv) Determination of the positions Q, R and S by second-order approximation

$$x_Q^{k+1} = x - \left[\frac{2\Delta t}{\frac{1}{u_Q^k} + \frac{1}{u_P^k}} \right] \tag{22}$$

$$x_R^{k+1} = x - \left[\frac{2\Delta t}{\frac{1}{(u-a)_R^k} + \frac{1}{(u+a)_P^k}} \right] \tag{23}$$

$$x_S^{k+1} = x - \left[\frac{2\Delta t}{\frac{1}{(u-a)_S^k} + \frac{1}{(u-a)_P^k}} \right] \tag{24}$$

where the superscripts k and k+1 represent the iteration numbers.

(v) Determination of the fluid properties at Q, R and S by quadratic interpolation

Taylor's theorem was used with second-order accuracy. The resulting equation, whose derivation was given [1] is as follows:

$$p_Q^{k+1} = p_M \pm \frac{1}{2\Delta x}(p_L + p_N)(x_Q^{k+1} - x) + \frac{1}{2(\Delta x)^2}(p_L + p_N - 2p_M)(x_Q^{k+1} - x)^2 \tag{25}$$

The positive and negative signs are used if the position of Q is between M & N and L & M respectively. The superscripts k and k+1 represent the iteration numbers. Similarly as for the first-order approximation, the other fluid properties and the properties at the other positions were calculated by replacing p and Q in equation (25) by the other properties and positions respectively.

(vi) Calculation of ρ, u and p at position P using second-order approximation

The same equations as used in the first-order approximation, were used. The fluid properties used to calculate the coefficients of the equations were averaged between those at the newly established positions of Q, R and S and those previously calculated at point P.

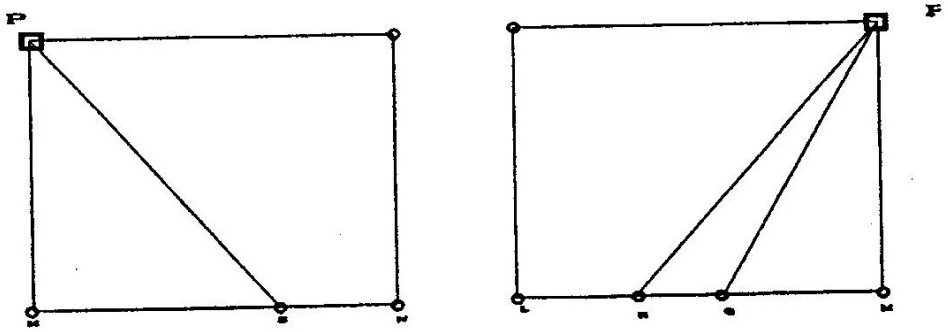
BOUNDARY CONDITIONS

In order to obtain solution at a boundary point, the number of additional equations required is the same as the number of characteristic curves lacking at the boundary point. Let us consider a pipe flow from an upstream boundary which is at a distance x_0 to a downstream boundary which is at a distance x_n. For calculation between two time levels, the meshes next to both boundaries are represented in Fig. 2. The characteristic curves defined in Fig. 1 are superimposed on the boundary meshes. The requirement is to find a solution at point P in Fig. 2(a) and (b). It is assumed that u is positive in the downstream direction and also the flow is subsonic at both the boundary points. The same procedure as for the interior points is used to calculate the solution at point P, but in this case the number of characteristics and hence the number of equations is less than the number of unknowns i.e. two equations less in Fig. 2(a) and one equation less in Fig. 2(b). In order to obtain a unique solution at the boundary points, the missing equations must be replaced by specifications for some of the dependent variables.

Boundary conditions which are commonly used are constant p, constant mass flow rate, constant T, the prescription of p as a function of t and the prescription of u as a function of t. The different possible boundary conditions and their solutions have been discussed[1].

VALIDATION OF THE COMPUTER MODEL

The problem of obtaining suitable experimental data for validation of computer models for modelling of linebreak in high-pressure gas pipelines was discussed [13] and [1]. Four sets of full-scale natural gas pipeline rupture experimental data on pipe sections of varying lengths, diameters and operating conditions were used to validate the computer model predictions. Only two of these sets of test data i.e. the Foothills[14] and the API [15] test data, are presented in this paper. The main purpose of the Foothills tests test was to examine the effect of gas composition on the fracture behaviour of the pipe. Short lengths of a total of 243m and diameters of approximately 1.2 and 1.4m were charged with natural gas of known composition and pressurised to between 74 and 87 barA. Fracture was initiated at the centre of the test section by detonating an explosive cutter. Although the data can be used to some extent to validate computer models for linebreak analysis, it was not intended for that purpose. The major reason which makes this data less suitable for validating linebreak models is the fact that the fracture was designed to propagate along the axial direction of the pipe covering some considerable lengths. This makes it difficult to model the break boundary, especially using this model where the break boundary is assumed to be fixed in the x-t plane. Only two test results, namely NABTF1 EAST and NABTF7 WEST were selected



(a) Upstream Boundary (b) Downstream Boundary

Fig. 2 Solution at boundary points

for validation of the computer model. The result NABFT1 EAST were used to validate the model for flow reversal in the downstream section of the broken pipe and the NABFT7 WEST results were used for the upstream

section. Results produced by the computer model are summarised in Figs. 3 to 6, together with the experimental results. A grid spacing of $\Delta x = 0.1$m and $\Delta t = 0.0001$s, at the broken end and a variable grid spacing were used.

The API test data is a full-scale experimental data. The tests were carried out using an existing 168.3mm outside diameter pipeline, which was typical of sour gas pipelines in the province at pressures of 6.9 and 3.45 MPa respectively. The test section was approximately 4.0 km long. It was ruptured at the mid point. Another test was performed on a 323.9 mm outside diameter and approximately 7.1 km long pipeline. Also in this test the pipeline was pressurised to 6.9 MPa pressure and ruptured at the middle. Due to the long run times required, a coarse grid spacing was used in order to reduce the CPU time. Agrid spacing of Δx=10 m and Δt=0.01 s and a variable grid spacing were used. The data for the latter test (APIT3), and the corresponding prediction from the computer model, are presented in Figs. 7 and 8.

DISCUSSION OF VALIDATION RESULTS

Simulation results produced in this study using the method of characteristics compare very well with the experimental data.The final pressures calculated for the Foothill tests were slightly higher than the experimental values. The reason for this is that the theoretical models did not account for the crack propagation along the length of the pipe. Results form theAPI data, which involve relatively long pipes were more consistent and satisfactory A good agreement was obtained between the experimental data and the prediction results, even with the big grid size used. The major weakness of the data is that they do not contain sufficient information about the gas used, some specifications of the testing system and accuracy of measurements recorded. The values produced with the first-order method compared much better with the experimental data than those obtained using the second-order method. The values of mass flow rates produced by the first-order method of characteristics are slightly lower than the experimental values. The reasons for this discrepancy are errors in calculating the gas density in the test results and the big grid spacing used. Problems of numerical instability, accuracy of results and singularity such as those encountered by Flatt [16] and Tiley [10], do not exist with this model.

Three main categories of error in validating the computer model are

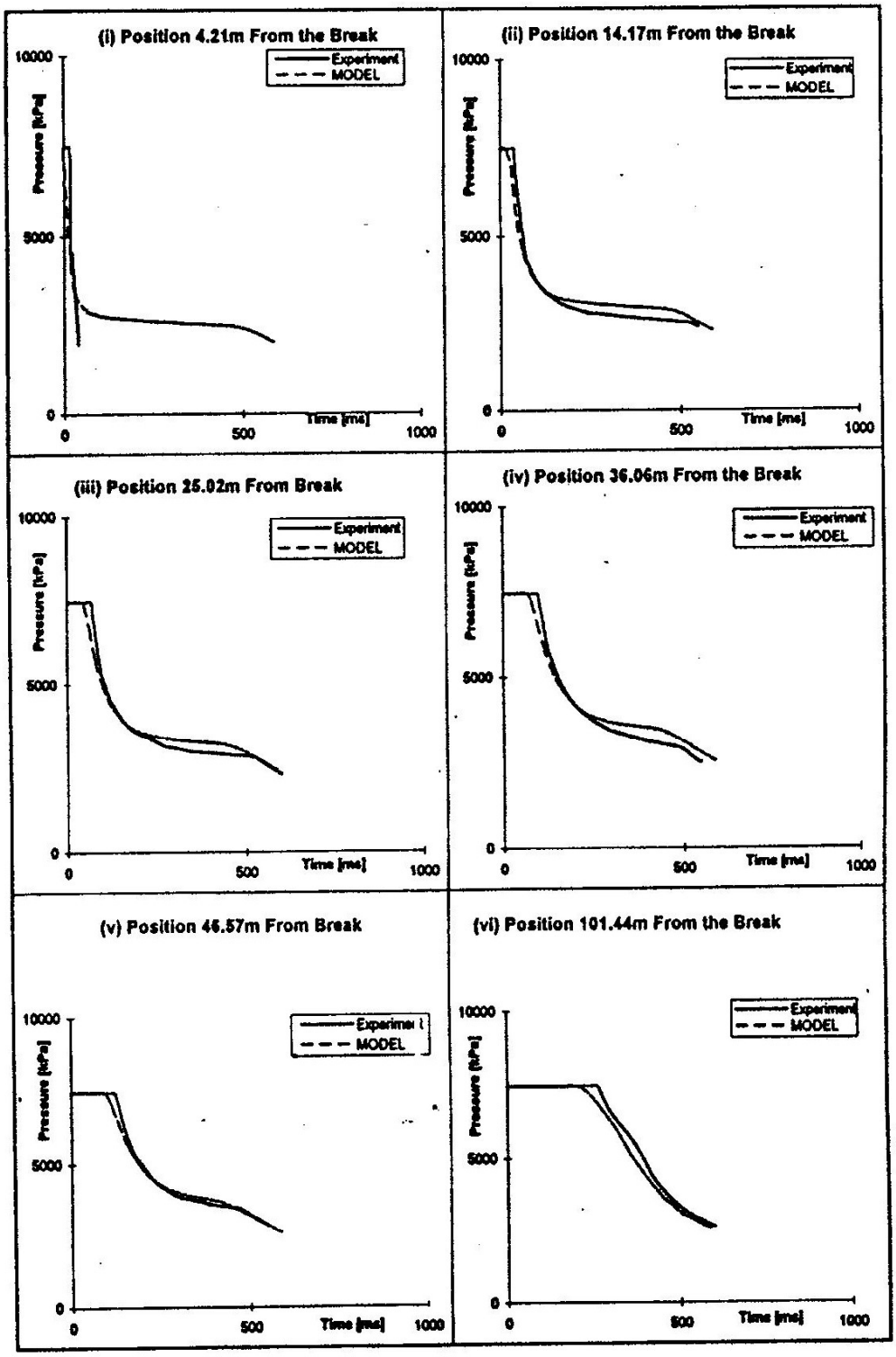


Fig.3: p-t curves for Foothills test NABTF1 east comparison with the model prediction

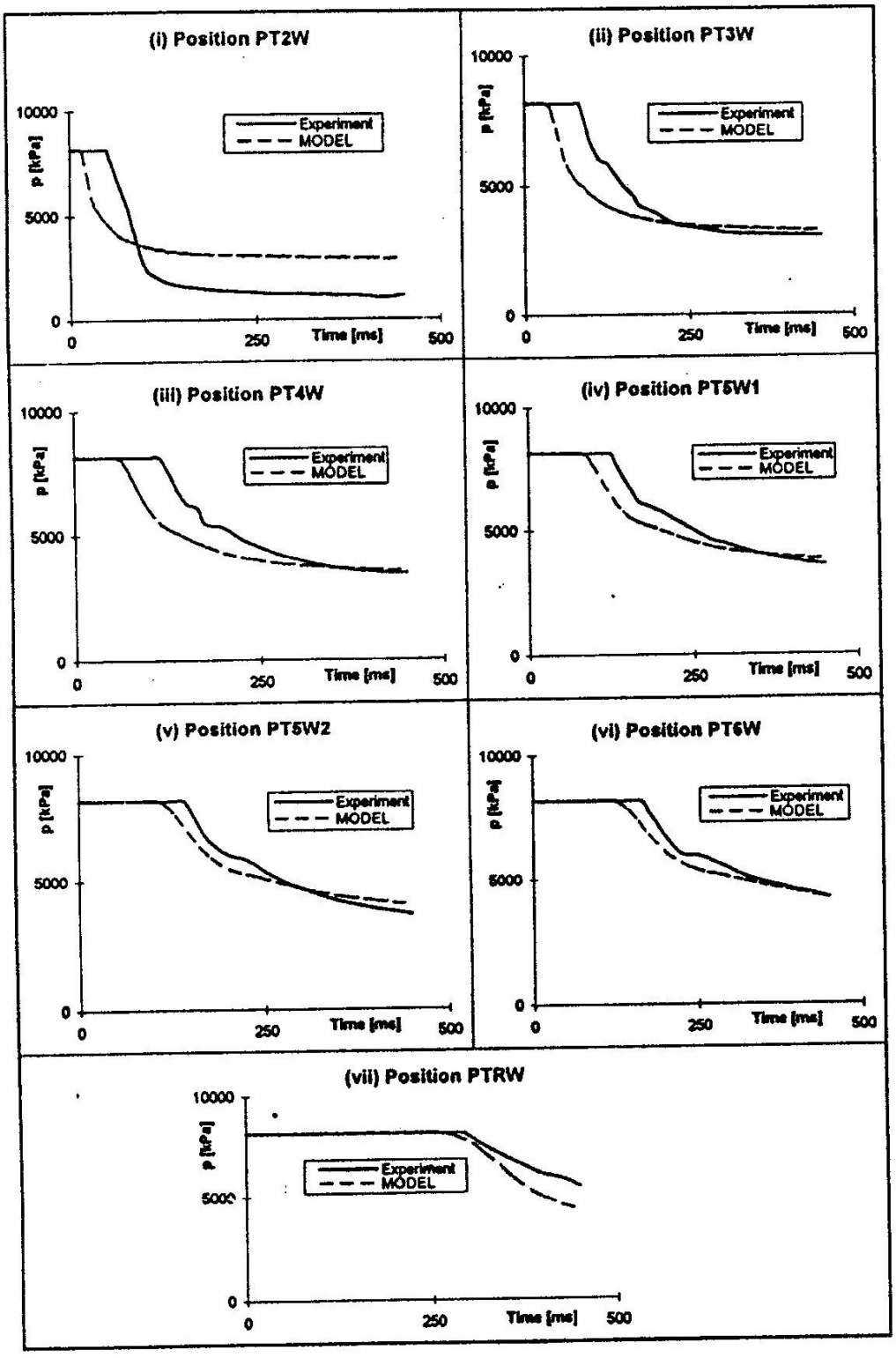


Fig. 4 p-t Curves for Foothills test NABTF7 west comparison with model prediction

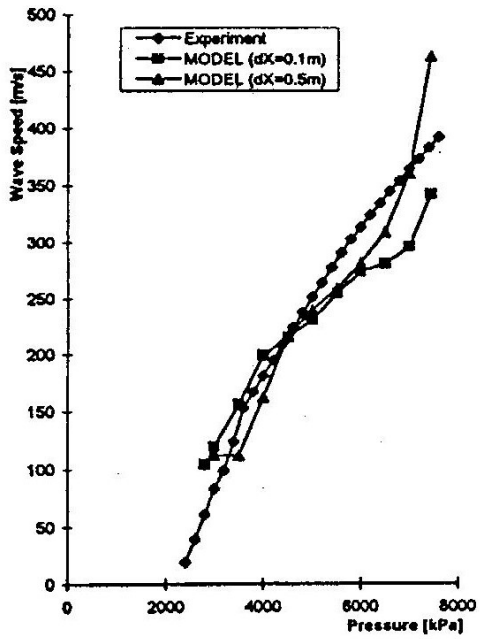


Fig. 5: Wave speed for NABTF1 east

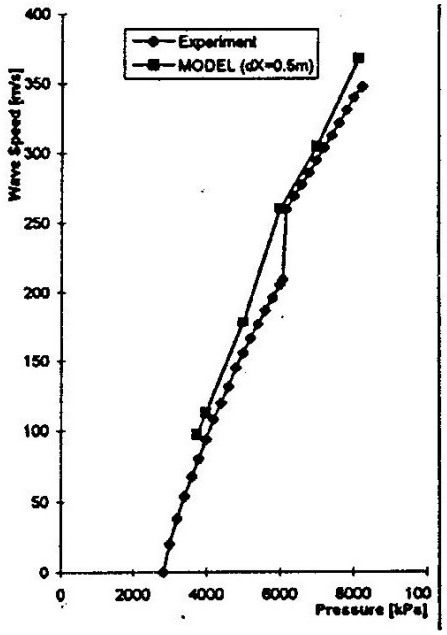


Fig. 6 Wasve speed for NABF7 west

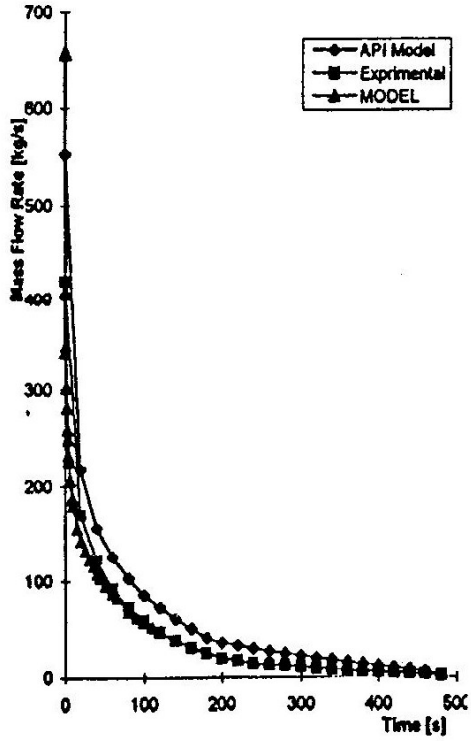


Fig. 7: Mass flow rate for API test

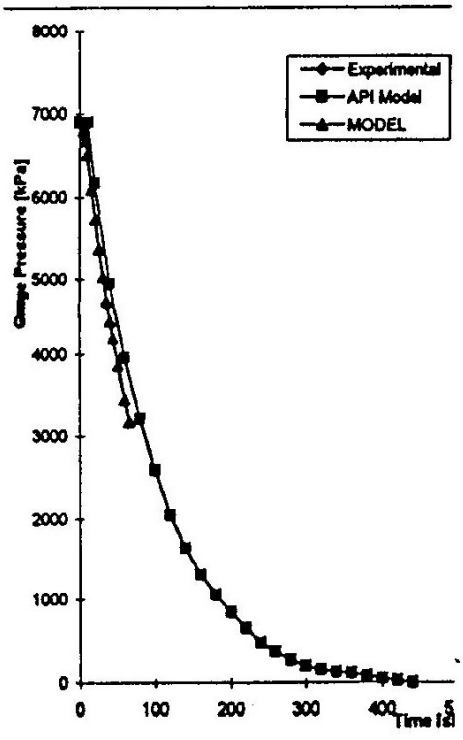


Fig. 8: Pressure profile for API test

calibration, measurement and recording of experimental data errors; errors due to assumptions and simplifications made in the basic theoretical equations; and errors inherent in the numerical modelling procedure.The error in experimental data also includes the error in converting graphical data into numerical data. No error estimate was provided with the experimental data. The error due to assumptions and simplifications in the theoretical model were greatly minimised. This model still contains some simplifications such as one-dimensional flow, single-phase flow, non-elastic pipe, no FSI and neglecting minor losses. However, the error introduced by these assumptions should be minimal in the case of straight horizontal pipes with constant cross-section area. The errors in estimating the friction factor and heat transfer have been minimized by using flow dependent values, which are specific for each calculation step. It was not possible to establish the magnitude of the accuracy of the computer model developed in this study, with certainty, because of the poor quality of the experimental data which has been used for validation. However, in most cases the predicted results are in good agreement with the experimental data.

CONCLUSIONS

A theoretical model has been developed for analysis of the transient flow following linebreak in high-pressure natural gas pipelines. The basic equations of flow are based on the gamma delta method. The three partial differential equations of flow were derived for unsteady quasi-one-dimensional flow of a real gas through a non-rigid non-constant cross-sectional area pipe. The QUANT software for thermodynamic and transport properties of the fluids was used. The flow dependent explicit equation of Chen [3] was used to calculate frictional force. The heat transfer through the pipe was calculated using a formula which is based on the adiabatic wall temperature and recovery factor. The heat transfer was also flow dependent; and the calculation procedure included both pipes exposed to the atmosphere and buried pipes. A non-uniform grid spacing was used, in order to be able to handle long pipelines, to produce stable results and to also adequately model the physical behaviour of the gas, following a rupture. A possibility of modelling the flow reversal in the section of the pipeline downstream of the break was provided. The theoretical transient analysis model was developed into a PC based computer code using the C programming language.

The transient analysis models, based on the method of characteristics, produced results which are in agreement with experimental data. A PENTIUM P75 was just adequate to run the transient analysis programmes. The computer model, is very stable numerically and it does not sufer singularity problems. The model contains the additional feature of being able to model heat transfer for cases where the pipeline is buried under water, ground or any other medium whose thermal conductivity and also the depth of the pipe in the medium are known. The first- and second-order methods produce results that are very close. The first-order method is over two times faster than the second-order method in computation speed and in some cases the former method handles the boundary conditions better than the latter method. At positions which are further away from the break, the first-order method seems to produce better results than the second-order method.

NOMENCLATURE

A	=	Cross-section area of pipe
a	=	Wave speed
C_n	=	Courant number
d	=	Pipe diameter
f_D	=	Darcy's friction factor
g	=	Gravitational acceleration
h	=	Specific enthalpy of gas
L	=	Length of pipeline
M_a	=	Mach number
p	=	Static pressure of gas
Pr	=	Prandtl number
t	=	Time
u	=	Flow velocity of gas
x	=	Horizontal distance along the pipe

Greek Symbols

λ	=	Gradient of the characteristic lines
γ_S	=	Isentropic gamma coefficient
Δ	=	Small change in the quantity
δ_S	=	Isentropic delta coefficient

θ	=	Angle of inclination of pipe to horizontal
μ	=	Coefficient of dynamic viscosity
ρ	=	Density of gas
ψ	=	Conical angle of the pipe
Ω	=	Heat flow into the pipe per unit length of pipe per unit time
ω	=	Frictional force per unit length of pipe

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