PREDICTION OF THE RESPONSE OF RECTANGULAR CLADDING PANELS TO WIND LOADING IN THE TIME DOMAIN

Ambrose A. Shirima
Department of Structural Engineering
University of Dar Es Salaam

The analysis and subsequent design of cladding panels is traditionally based on quasi-static calculation of the wind loads. Pressure coefficients, which are derived from wind tunnel measurements, and a design wind speed, are used to compute the wind loading. The codes of practice give the pressure coefficients and the guidelines of deriving the design wind speed from factors, which are specific to the site namely basic wind speed, topography, ground roughness and height above ground. The concept of design wind speed is simple but it overlooks the resonant vibrations caused by the wind. The paper discusses a procedure of incorporating the time and spatial variations of the wind pressure distribution in the computation of the response of rectangular cladding panels to wind loads. The panel is subdivided into several portions. For each portion, the Davenport spectrum for wind velocity and the Shinozuka procedure for generating stationary random processes with specified spectral density are used to generate a random function, which simulates the fluctuating wind velocity at that portion. For the purpose of simulating the spatial variations in the wind pressure distribution on the cladding panel, the random functions differ from portion to portion. The correlation of the wind velocities at two different portions on the panel is described with the help of a correlation matrix. The finite element method is used to formulate the equations of motion, which are finally solved using the Newmark numerical integration procedure. Analysis of various cladding panels using both procedures showed that the maximum displacement obtained by the dynamic procedure is more than twice the maximum displacement obtained by the quasi-static procedure. The results of the analysis indicate that there is a need to base the design and analysis of cladding panels on a dynamic procedure. The results also indicate that to assess the performance of existing cladding panels, one should use a dynamic analysis because the quasi-static static analysis can lead to wrong conclusions.

Keywords: *mean square value, wind spectrum, cross correlation, correlation matrix, numerical integration*

INTRODUCTION

Wind loading is the main force, which acts on cladding panels. Therefore accurate prediction of the performance of such panels to wind action depends very much on the mathematical model used to describe the wind forces.

Currently the codes of practice give guidelines of computing the wind pressure if the terrain roughness, topography and height above ground of the site are known. For example in accordance with Eurocode 1: Part 2.4 the wind pressure acting on a surface is obtained from :

$$w_e = \frac{\rho}{2} v_{ref}^2 \cdot c_e(z) \cdot c_p \tag{1}$$

where

w_e = Wind pressure acting on the surface
ρ = air density
$vref$ = reference wind velocity
$ce(z)$ = exposure coefficient accounting for the terrain and height above ground
z = reference height
c_p = pressure coefficient

It is to be noted that the wind pressure given by Equation [1] is static. However, wind is known to vary randomly both in space and in time. Therefore Equation [1] can not give a realistic estimate of the response of the cladding panels to wind loading because it does not cover the vibrations, which can be caused by the wind.

The paper discusses an analytical procedure of predicting realistically the performance of cladding panels to wind loading. The analysis is done in the time domain and it is based on generated random series, which simulates the wind velocity.

The panel is subdivided into several portions. Using the Davenport spectrum (Davenport 1961) of wind velocity fluctuations, the Shinozuka procedure (Shinozuka 1971) of generating stationary random processes with specified spectrum is used to generate a random time function, which represents the fluctuating wind velocity at each portion. The random functions vary from portion to portion in order to simulate the spatial variations of the wind velocity.

THE AERODYNAMIC FORCES ON THE CLADDING PANEL

The wind forces on the cladding panel can be expressed in the most general form as:

$$F=F_D+F_a \tag{2}$$

where F_D = Drag force component of the wind action

F_a = Self excited forces that arise from Aero elastic phenomenon

In this study the self-excited forces were neglected. The drag force is given by (August *et al.* 1986):

$$F_D(t)=\frac{1}{2}\cdot\rho\cdot C_D\cdot A\cdot\left(\bar{v}+v(t)-\dot{w}(t)\right)^2+C_M\cdot V_{air}\cdot\rho\cdot\dot{v}_R(t) \tag{3}$$

where: ρ = density of air, A = Area of the cladding panel, V_{air} = Volume of air participating in the motion of the panel, $\bar{v}$ = mean wind velocity, $v(t)$ = fluctuating component of the wind velocity, $\dot{w}$(t) = velocity of the panel due to wind action, v_R(t) = relative velocity ($\bar{v}+v(t)-\dot{w}(t)$), $\dot{v}_R$(t) = relative acceleration, C_D = drag coefficient and C_M = virtual mass coefficient.

Taking into account the small density of air and the volume of air participating in the motion of the panel, the second term of Equation [3] has been neglected in this study. Furthermore, one can assume that $(v(t)\approx\dot{w}(t)) << \bar{v}$ (Sockel, 1984). With these assumptions, Equation [3] takes the form:

$$F_D(t)=\frac{1}{2}\cdot\rho\cdot C_D\cdot A\cdot\bar{v}^2+A\cdot\rho\cdot C_D'\cdot\bar{v}\cdot v(t)-A\cdot\rho\cdot C_{da}\cdot\bar{v}\cdot\dot{w}(t) \tag{4}$$

where C_D = Drag coefficient for the mean wind velocity

C_D' = Drag coefficient for the fluctuating component of the wind velocity

C_{da} = Coefficient for the aerodynamic damping force

Neglecting the aerodynamic damping force the aerodynamic force on the panel becomes:

$$F_D(t)=\frac{1}{2}\cdot\rho\cdot C_D\cdot A\cdot\bar{v}^2+A\cdot\rho\cdot C_D'\cdot\bar{v}\cdot v(t) \tag{5}$$

Equation [5] indicates the following:

- The time histories of the wind velocity are among the essential inputs in the analysis of structures under wind action.
- It is convenient to separate the wind velocity into two components, namely a mean wind velocity and deviations of the actual wind velocity from the mean. Various researchers have made this has observation. For example according to (Simiu *et al.* 1986) the wind velocity can be expressed as:

  $$V(t)=\bar{v}+v(t) \tag{6}$$

  where $V(t)$ = the wind velocity, $\bar{v}$ = mean velocity and $v(t)$ = deviations from the mean
- Since the wind velocity is random both in space and time, the basis of the input data for the wind velocity can only be the statistical characteristics of wind because it is not possible to get a single time history, which can

reproduce the spatial and temporal variations in the wind velocity.

WIND CHARACTERISTICS

The statistical characteristics of the wind velocity, which form a basis for the dynamic analysis of cladding panels under wind loading are the mean, the mean square value, the auto correlation function, the power spectral density (power spectrum), and cross correlation function.

THE MEAN WIND VELOCITY

The mean wind speed is the basis of static analysis and design of structures. It varies with the height as represented by the mean wind velocity profile. According to Davenport *et al.* 1988), (Rauscheweyh 1982) and (Zuranski 1981) the mean wind velocity profile can be represented by the following empirical power law.

$$\frac{\bar{v}_z}{\bar{v}_{10}} = \left[\frac{z}{10}\right]^{\alpha} \qquad (7)$$

where $\bar{v}_z$ = mean wind velocity at a Height z m above the ground.

$\bar{v}_{10}$ = reference wind velocity at a height of 10 m above the ground in a free field

z = height above ground in meters

α= roughness coefficient (α = 0.16 for open country, α = 0.28 for woods, villages and towns, α = 0.40 for city centres) (Davenport *et al.* 1988).

It is to be noted that the parameters α in Equation [7] depend on the roughness of the terrain. Thus the wind velocity profile for an open country is different from the wind velocity profile for the central areas of big cities, where there are many windbreaks. The reference velocity is usually defined for a 10-minute-mean value and a return period of 50 years (Rauscheweyh 1982).

Figure [1] shows a vertical profile of mean wind speed for parts of Dar Es Salaam with many wind breaks e.g. the city centre. The figure is based on a basic wind speed of 27m/s, which is reported in (Lwambuka 1991).

THE MEAN SQUARE VALUE

In the study of wind effects on structures the wind can be treated as a stationary random process (Davenport 1961). Therefore, the mean square value of the velocity fluctuations can be approximated by the time average (William 1988):

$$\overline{v^2} = \frac{1}{T}\int_0^T v^2(t)dt \qquad (8)$$

THE POWER SPECTRAL DENSITY

The power spectral density describes the frequency composition of a random variable. According to (William 1988) the power spectral density of a random variable *x(t)* is defined as the mean-square response of an ideal narrow-band filter to *x(t)*, divided by the bandwidth Δf of the filter in the limit as $\Delta f \rightarrow 0$ at the frequency f(Hz). Thus, the power spectral density of the wind velocity fluctuations can be defined as:

$$S_v(f) = \lim_{\Delta f \rightarrow 0} \frac{\overline{v_{\Delta f}^2}}{\Delta f} \qquad (11)$$

From Equation [11] it follows that the mean square value of the wind velocity fluctuations is equal to the sum of the power spectral components over the entire frequency range i.e.:

$$\overline{v^2} = \int_0^{\infty} S_v(f)df \qquad (12)$$

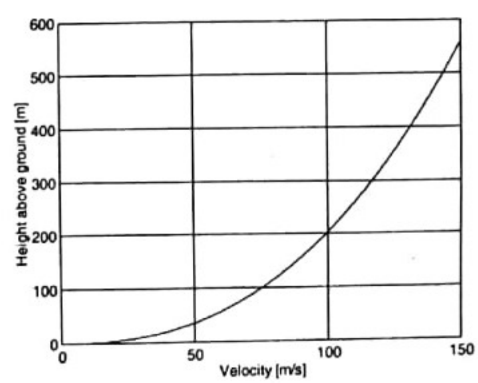

Figure 1: Profile of the Mean Wind Speed for Parts of Dar es Salaam with Many Wind Breaks

Various spectra for the wind velocity fluctuations have been proposed in the literature (Davenport 1961), (Simiu *et al.* 1986), Peil *et al.* 1997), Kaimal *et al.* 1972) and (Harris 1971). The present work has used Davenport Spectrum (Davenport 1961)

$$S(f) = \frac{\kappa \cdot \bar{v}_{10}}{f}\left(\frac{4 \cdot x^2}{\left(1 + x^2\right)^{\frac{4}{3}}}\right) \tag{13}$$

Where f = frequency

$\bar{v}_{10}$ = velocity at standard reference height of 10m

κ = drag coefficient referred to mean velocity at 10m

$x = \dfrac{1200f}{\bar{v}_{10}}$

THE CROSS CORRELATION FUNCTION

The spatial variations of the wind velocity at two different points i and j on the panel can be described by the cross correlation function, which is defined as:

$$R_{vivj} = \lim_{T \to 0} \frac{1}{T} \int_0^T v_i(t) \cdot v_j(t+\tau)dt \tag{14}$$

Where

v_i , v_j = Fluctuating component of the wind velocity at points i and j respectively

τ = delay (William 1988), (Rauscheweyh 1982):

According to (Davenport 1993) and (Saul *et al.* 1976) the cross correlation of wind velocity fluctuations can be expressed as a function of separation, Δr, of the two points j *and* i. When expressed as a function of separation it takes on the form (Davenport 1993):

$$R_{vivj} = e^{-\frac{\Delta r}{Lu}} \tag{15}$$

in which Δr = Separation, and Lu = Length scale describing the spatial extent of the wind gust. According to (Davenport 1993) Lu = 30m-60m approximately.

ANALYSIS OF THE STRUCTURAL RESPONSE OF CLADDING PANEL TO WIND LOADING

In this study the Finite Element Method (FEM) was used to analyse the response of cladding panels to wind loading. The analysis involved the following steps:

- Subdividing the cladding panel into several rectangular elements
- Generation of artificial wind velocity histories, which have the spectral and cross-correlation properties of the natural wind, at the centre of each element.
- Determination of the aerodynamic wind pressure on the panel
- Formulation of the equations of motion of the cladding panel
- Solving the equations of motion using the Newmark step-by-step numerical integration procedure to get the structural response.

GENERATION OF TIME HISTORIES OF WIND VELOCITY FLUCTUATIONS

Using the Shinozuka procedure (Shinozuka 1971), time histories of the wind velocity fluctuations were generated in three steps namely:

- Generation of N uncorrelated processes $Y=\{Y_1(t)\ Y_2(t)\ldots\ldots Y_i(t)\ldots\ldots\ldots Y_N(t)\}$, where N is the number of elements, which will be used to analyse the cladding panel and $Y_i(t)$ represents the time history of the wind acting at the centre of element i.
- Determination of the target correlation matrix for the cladding panel
- Transforming the uncorrelated processes to correlated processes

GENERATION OF UNCORRELATED PROCESSES $Y_i(T)$

Each process $Y_i(t)$ is simulated by the substitution

$$Y_i(t) = \sum_k^M A_k \cos(2\pi f_k t + \varphi_{ik}) \qquad (16)$$

where

i = indicates centre of element i of the finite element model of the cladding panel

$Y_i(t)$ = Random process representing the wind velocity.

t = time.

M = Number of intervals into which the frequency range of the wind velocity spectrum has been divided.

k = the number of a frequency interval.

f_k = the value of the frequency in the centroid of interval k in cycles/second as illustrated in Figure 2 below.

A_k = Amplitude of the harmonic component with the frequency η_k.

φ_{ik} = random phase angles, uniformly distributed between 0 and 2π.

From the requirement that the statistical properties of the random processes $Y_i(t)$ should be equal to those of the natural wind, the spectrum of the natural wind together the definitions of the mean square value of the wind velocity fluctuations in the frequency domain and in the time domain, are used to compute the unknown amplitudes A_k

In the time domain the mean square value is given by Equation [4] as:

$$\overline{Y_i^2} = \frac{Lim}{T \to 0}\frac{1}{T}\int_0^T \left(\sum_k^M A_k \cos(2\pi f_k t + \varphi_{ik}) \right)^2 dt \qquad (17)$$

But $A_\alpha \cos(2\pi f_\alpha t + \varphi_{i\alpha})$ and $A_\beta \cos(2\pi f_\beta t + \varphi_{i\beta})$ are independent for α≠β. Therefore

$$\overline{Y_i^2} = \frac{Lim}{T \to 0}\frac{1}{T}\int_0^T \sum_k^M A_k^2 \cos^2(2\pi f_k t + \varphi_{ik})\, dt = \sum_k^M \frac{A_k^2}{2} \qquad (18)$$

In the frequency domain the mean square value is given by Equation [12] as:

$$\overline{Y_i^2} = \int_0^\infty S_v(f)df \approx \sum_k^M S_v(f_k)\cdot \Delta f \qquad (19)$$

From Equations [18] and [19] one gets the following expression for calculation the amplitudes A_k of the harmonics present in the wind.

$$\sum_k^M \frac{A_k^2}{2} = \sum_k^M S_v(f_k)\cdot \Delta f \qquad (20)$$

From Equation [20] it follows:

$$A_k = \sqrt{2\cdot S_v(f_k)\cdot \Delta f} \qquad (21)$$

GENERATION OF CORRELATED PROCESSES $\underline{V}(T)$

The uncorrelated processes $\underline{Y}(t)$ are transformed to correlated processes $\underline{v}(t)$ by a linear transformation:

$$\underline{v}(t) = \underline{H} \bullet \underline{Y}(t) \qquad (22)$$

Where

$\underline{v}(t) = [v_1(t) \quad v_2(t) \ldots v_i(t) \ldots v_N(t)]^T$ = Correlated wind velocity fluctuations

$\underline{Y}(t) = [Y_1(t) \quad Y_2(t) \ldots Y_i(t) \ldots Y_N(t)]^T$ = The uncorrelated processes generated above

$\underline{H}$ = an $N \times N$ transformation matrix

The elements of matrix $\underline{H}$ are obtained with the help of Equations [14] and [15]:

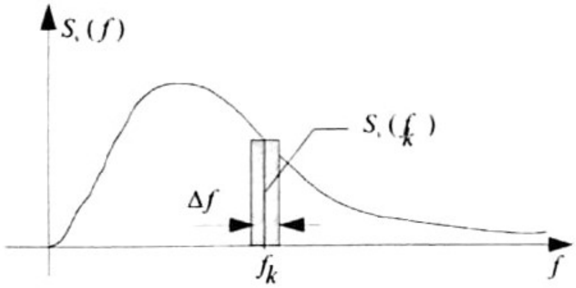

Figure 2: The Wind Spectrum as Applied in the Simulation of Time Histories

Using Equation [14] the correlation matrix of the wind velocities at different points on the panel is given by:

$$\underline{\boldsymbol{R}} = \lim_{T\to 0}\frac{1}{T}\int_0^T \underline{v}\cdot\underline{v}^T dt = \lim_{T\to 0}\frac{1}{T}\int_0^T \underline{H}\cdot\underline{Y}\cdot\underline{Y}^T\cdot\underline{H}^T\cdot dt$$

or

$$\underline{\boldsymbol{R}} = \underline{H}\cdot\left(\lim_{T\to 0}\frac{1}{T}\int_0^T \underline{Y}\cdot\underline{Y}^T\cdot dt\right)\cdot\underline{H}^T \qquad (23)$$

Since the processes $Y_i(t)$ are independent it follows that $\underline{\boldsymbol{R}}_{YY} = \lim_{T\to 0}\frac{1}{T}\int_0^T \underline{Y}\cdot\underline{Y}^T\cdot dt$ is a diagonal matrix and its (i,i) value is the variance of the i^{th} process $Y_i(t)$. This variance is given by Equations [10] and [19] i.e.

$$\sigma_i^2 = \overline{Y_i^2} = \int_0^\infty S_v(f)df \approx \sum_k^M S_v(f_k)\cdot\Delta f \qquad (24)$$

Since the same spectrum $S_v(f)$ was used to generate the processes $Y_i(t)$ $(i = 1 \ldots\ldots N)$ it follows that $\sigma_1^2 = \sigma_2^2 = \sigma_i^2 = \ldots\ldots\ldots = \sigma_N^2$ =constant= σ_v^2 where σ_v^2 is the variance of the wind velocity fluctuations. Therefore the correlation matrix of the correlated wind velocity fluctuation $\underline{v(t)}$ takes the form:

$$\underline{\boldsymbol{R}} = \sigma_v^2\cdot\underline{\boldsymbol{H}}\cdot\underline{\boldsymbol{I}}\cdot\underline{\boldsymbol{H}}^T \qquad (25)$$

The elements of the transformation matrix $\underline{\boldsymbol{H}}$ are obtained from the requirement that the matrix should be equal to the cross correlation matrix of the natural wind, which is given by Equation [15] i.e.

$$R(i,j) = e^{-\frac{r_{i,j}}{Lu}} \qquad (26)$$

Where

$R(i,j)$ = cross correlation of the wind velocities at points i and j on the panel

$r_{i,j}$ = distance between the two points

Lu = Length scale as defined in Equation [15]. (In this study a value of Lu = 45m was used)

For convenience purposes a triangular matrix is chosen for the transformation matrix. In that case the elements of the matrix are easily obtained recursively.

Figure 3 shows a time history of the generated wind. In the said figure, a mean velocity of 30m/s has been added to the generated wind velocity fluctuations. The spectrum of the generated wind is compared with the spectrum of the natural wind in Figure 4.

Figure 4 shows that the spectrum of the generated time history is very close to the spectrum of the

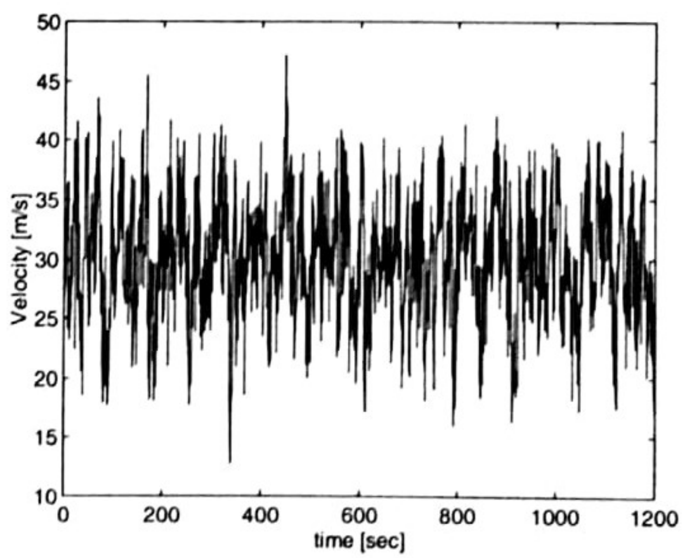

Figure 3: Time History of Generated Wind

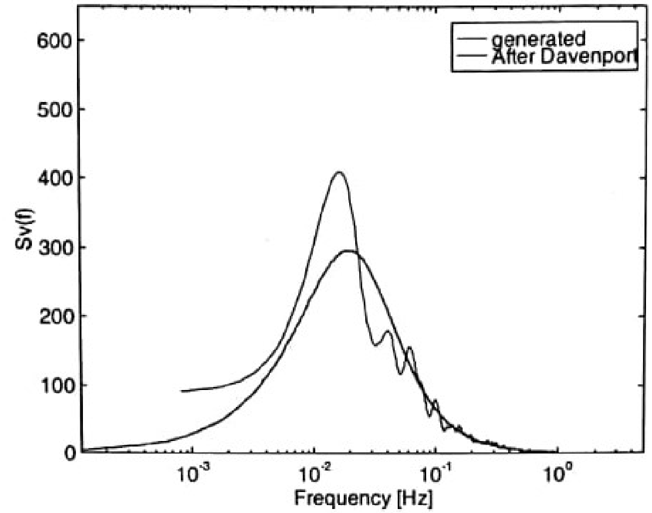

Figure 4: Spectrum of the Generated Wind is compared with the Spectrum of Natural Wind

natural wind. Therefore it can be used without a lot of reservations in the time domain analysis of cladding panels under wind loading.

DETERMINATION OF THE AERODYNAMIC WIND PRESSURE ON THE PANEL

The wind pressure is obtained from Equation [5] as:

$$P(t) = \frac{1}{2} \cdot \rho \cdot C_D \cdot \bar{v}^2 + \rho \cdot C'_D \cdot \bar{v} \cdot v(t) \qquad (27)$$

whereby *P(t)* is the total wind pressure on the panel. Equation [27] can be written as:

$$P(t) = \bar{p} + p(t) \qquad (28)$$

where $\bar{p} = \frac{1}{2} \cdot \rho \cdot C_D \cdot \bar{v}^2$ is the mean wind pressure and $p(t) = \rho \cdot \bar{v} \cdot C'_D \cdot v(t)$ is the fluctuating wind pressure. From $\bar{p} = \frac{1}{2} \cdot \rho \cdot C_D \cdot \bar{v}^2$ one can derive the following expression for the fluctuating wind pressure:

$$p(t) = \rho \cdot \bar{v} \cdot C'_D \cdot v(t) = 2 \cdot \frac{\bar{p}}{\bar{v}} \cdot \frac{C'_D}{C_D} \cdot v(t) =$$

$$2 \cdot \frac{\bar{p}}{\bar{v}} \cdot \chi_a \cdot v(t) \qquad (30)$$

where $\chi_a = \frac{C'_D}{C_D}$ is defined as aerodynamic magnification function (Rauscheweyh 1982). According (Sockel 1984) χ_a=1 when determining local wind effects. On these grounds a value of χ_a=1 was used in this study to obtain the following expressions for the wind pressure on the cladding panel:

$$\bar{p} = \frac{1}{2} \cdot \rho \cdot \bar{C}_p \cdot \bar{v}^2 \text{ and } p(t) = 2 \cdot \frac{\bar{p}}{\bar{v}} \cdot v(t) \qquad (30)$$

Where $\bar{C}_p$ = pressure coefficient obtainable from the codes of practice, $\bar{v}$ is the mean wind speed appropriate to the region where the panel is, $v(t)$ is the fluctuating component of the wind velocity that was generated as described above.

FORMULATION OF THE EQUATIONS OF MOTION OF THE CLADDING PANEL

Formulation of the equations of motion was based on the Hamilton's principle, which may be stated as (Link 1986)

$$\int_{t_1}^{t_2} \delta(\Pi^i + \Pi^e - \Pi^k) dt + \int_{t_1}^{t_2} \delta Wncdt = 0 \qquad (31)$$

where Π^i = Strain energy of the system

Π^c = potential energy of conservative external forces (in this study wind load).

Π^k = Total kinetic energy of the system

W_{nc} = work done by non conservative forces (in this study only damping force)

δ = variation taken during a specified time interval.

Expressing Π^i, Π^c, Π^k and W_{nc} in terms of the appropriate structural properties gives:

$$\Pi^i = \frac{1}{2} U_t^T \cdot K \cdot U_t$$

$$\Pi^c = -U_t^T \cdot \mathbf{P}$$

$$\Pi^k = \frac{1}{2} \dot{U}_t^T \cdot M \cdot \dot{U}_t$$

$$W_{nc} = U^T \cdot F_D \tag{32}$$

where $\mathbf{U}_t$ = Vector of the nodal displacements at time t

K = Stiffness matrix of the cladding pane

M = Mass matrix of the cladding panel

P = Vector of the equivalent nodal wind loads

$\mathbf{F_D}$ = Vector of damping forces

Substituting Equation [32] into Equation[31] gives

$$\int_{t_1}^{t_2} \left\{ \delta\left(\frac{1}{2} U_t^T K U_t - \frac{1}{2} \dot{U}_t^T M \dot{U}_t - U_t^T P \right) + \delta U_t^T F_D \right\} dt = 0 \tag{33}$$

The variation in Equation[33] and the subsequent partial integration of $\int \dot{U}_t^T \cdot M \cdot \dot{U}_t \cdot dt$ leads to

$$M\ddot{U}_t + F_D + KU_t = P \tag{34}$$

In this study the damping forces were assumed to be proportional to the velocity of the panel. With this assumption Equation [34] takes the final form

$$M\ddot{U}_t + C\dot{U}_t + KU_t = P_t \tag{35}$$

where $\boldsymbol{C}$ is a damping matrix, which was assumed to be equal to the Rayleigh damping. The Rayleigh damping has the form (Klaus-J rgen 1986):

$$\boldsymbol{C} = k_1 \boldsymbol{M} + k_2 \boldsymbol{K} \tag{36}$$

where k_1 and k_2 are constants, which are determined by two consecutive natural frequencies, whose damping ratios are known. They were computed by solving the following simultaneous equations:

$$k_1 + k_2 \omega_i^2 = 2 \cdot D_i \cdot \omega_i$$

$$k_1 + k_2 \omega_j^2 = 2 \cdot D_j \cdot \omega_j \tag{37}$$

with ω_i , ω_j = two consecutive natural frequencies

D_i, D_j = Damping ratios of the natural modes i and j.

The damping ratios used in this study were obtained from [20].

SOLUTION OF THE EQUATIONS OF MOTION OF THE CLADDING PANEL

The Newmark numerical integration procedure was used to solve the equations of motions. The procedure is based on the equations of motion at time $t+\Delta t$ and following finite difference relationships:

$$U_{(t+\Delta t)} = U_t + \dot{U}_t \cdot \Delta t + \left[(0.5-\beta) \cdot \ddot{U}_t + \beta \cdot \ddot{U}_{(t+\Delta t)} \right] \cdot \Delta t^2 \tag{38}$$

$$\dot{U}_{(t+\Delta t)} = \dot{U}_t + \left[(1-\delta) \cdot \ddot{U}_t + \delta \cdot \ddot{U}_{(t+\Delta t)} \right] \cdot \Delta t \tag{39}$$

where

Δt = time step

U_t, $U_{(t+\Delta t)}$ = vector of the nodal displacements at time t and at time $t+\Delta$

$\dot{U}_t$, $\ddot{U}_t$, $\dot{U}_{t+\Delta t}$, $\ddot{U}_{t+\Delta t}$ = velocity and acceleration vectors at time t and at time $t+\Delta$

δ, β = interpolation parameters

The equations of motion at time $t+\Delta t$ are given by:

$$M\ddot{U}_{t+\Delta t} + C\dot{U}_{t+\Delta t} + KU_{t+\Delta} = P_{t+\Delta t} \tag{40}$$

The accuracy and stability of the Newmark's method depends on the selected time step (Δt) as well as on the interpolation parameters (δ, β). The method is unconditionally stable if (Klaus-J rgen 1986)

$$\delta \geq \frac{1}{2} \quad \text{and} \quad \beta \leq 0.25(\delta + 0.5)^2 \tag{41}$$

Furthermore the time step Δt should be small enough to ensure that the response in all modes, which significantly contribute to the total structural response, is calculated accurately. In this study the following values were selected:

$\delta = 0.5$, $\beta = 0.25$ and $\Delta t = 0.025$s.

To obtain the displacements, velocities and accelerations at time $t+\Delta t$, Equations [38] and [39] were used to express $\ddot{U}_{t+\Delta t}$ and $\dot{U}_{t+\Delta t}$ in terms of the known displacements, velocities and accelerations at time t and the unknown displacement $U_{(t+\Delta t)}$.

From Equation [38]

$$\ddot{U}_{t+\Delta t} = \frac{1}{\beta \Delta t^2}\left(U_{t+\Delta t} - U_t\right) - \frac{1}{\beta \Delta t}\dot{U}_t - \left(\frac{1}{2\beta} - 1\right)\ddot{U}_t \tag{42}$$

or

$$\ddot{U}_{t+\Delta t} = b_0\left(U_{t+\Delta t} - U_t\right) - b_1\dot{U}_t - b_2\ddot{U}_t \tag{43}$$

where $b_0 = \frac{1}{\beta \Delta t^2}$, $b_1 = \frac{1}{\beta \Delta t}$, and

$$b_2 = \left(\frac{1}{2\beta} - 1\right)$$

Equation[42] is substituted into Equation[39] to get

$$\dot{U}_{t+\Delta t} = \frac{\delta}{\beta \Delta t}\left(U_{t+\Delta t} - U_t\right) - \left(\frac{\delta}{\beta} - 1\right)\dot{U}_t - \Delta t \cdot \left(\frac{\delta}{2\beta} - 1\right)\ddot{U}_t \tag{44}$$

or

$$\dot{U}_{t+\Delta t} = b_3\left(U_{t+\Delta t} - U_t\right) - b_4\dot{U}_t - b_5\ddot{U}_t \tag{45}$$

where $b_3 = \frac{\delta}{\beta \Delta t}$ $\quad b_4 = \left(\frac{\delta}{\beta} - 1\right)$ and

$$b_5 = \Delta t \cdot \left(\frac{\delta}{2\beta} - 1\right)$$

Equations [43] and [45] are now substituted into Equation [40] to give the following set of equations:

$$\left(M \cdot b_0 + C \cdot b_3 + K\right) \cdot U_{t+\Delta t} = P_{t+\Delta t} + M \cdot \left(b_0 U_t + b_1 \dot{U}_t + b_2 \ddot{U}_t\right) + C \cdot \left(b_3 U_t + b_4 \dot{U}_t + b_5 \ddot{U}_t\right) \tag{46}$$

Equation[46] is solved simultaneously to get the displacements at time $t+\Delta t$, which are finally substituted into Equation [43] and [45] to get respectively $\ddot{U}_{t+\Delta t}$ and $\dot{U}_{t+\Delta t}$. The initial conditions are normally assumed to be stationary i.e. $U_{t=0} = \dot{U}_{t=0} = \ddot{U}_{t=0} = 0$

NUMERICAL EXAMPLE

For the purpose of illustrating the difference between results obtained by the static analysis and those obtained by the dynamic procedure, a simply supported panel of size 3m x 0.9m with a thickness of 5 mm will be analysed. The parameters for the analysis are summarised in Table 1.

Figure 5 shows the results of the quasi-static analysis, which is based on the code of practice Eurocode I: Part 2.4. The figure shows that the maximum displacement at the centre of the panel is around 6mm.

Assuming stationary initial conditions, an artificial wind that was generated as discussed above was used to determine the dynamic response of the panel to wind loading. The dynamic analysis was performed for a total period of 60 seconds. The results of the dynamic analysis are shown in Figures 6 and 7. Figure 6 shows the time history of the displacement at the centre of the panel for various damping ratios while Figure 7 shows the deformed shape of a panel with a damping ratio of 0.1 % at various moments.

Figures 6 and 7 illustrate the following:

(i) The initial response of the panel to the wind loading is erratic and it is considerably higher

Table 1: Parameters for the Numerical Example

Material properties	Youngs Modulus of Elasticity	= 205 kN/mm^2
	Poisson's Ratio	= 0.3
	Density	= 78.5 kN/m^3
	Damping Ratio	= 0.1 %
Wind Characteristics	Mean Wind Velocity	= 30 m/s
	Density of air	=1.25 kg/m^3
	Pressure Coefficient ($\overline{C}_P$)	=0.8
	Wind Velocity at height of 10m ($\overline{v}_{10}$)	=30 m/s
	Drag coefficient ( )	=0.005
	Variance of Velocity Fluctuations	= 27.0 [m/s]^2

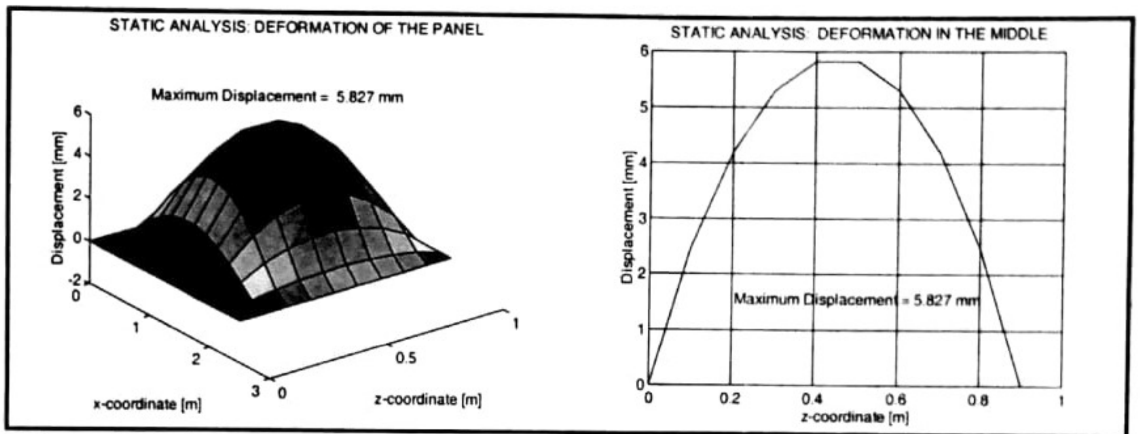

Figure 5: Static Response due to mean wind pressure

than the static response. For the damping ratios of 0.1% , 0.5% and 1% the maximum response is respectively 13.97mm, 13.16mm and 13.04 mm. Compared with the maximum displacement of 5.827 mm, which was obtained from the quasi static analysis, the maximum dynamic response is more than twice the maximum displacement given by the quasi static procedure.

(ii) In Figure 6 it is observed that the duration of the erratic behaviour depends on the damping present in the system. If the damping ratio is 0.1 % or less the panel behaves erratically for more than 60 sec but if the damping ratio is 1% the panel behaves erratically for only 10 sec.

(iii) The damping has no effect on the mean response of the panel to wind loading and has very little effects on the maximum response. It is seen in Figure 6 that the maximum response of a panel with a damping ratio of 0.1% is 13.97mm and the maximum response of a panel with a damping ratio of 1% is 13.04. Thus, the damping ratio increased ten times but the reduction in the maximum response is less than 1 mm.

CONCLUSIONS

The following conclusions are drawn based on the numerical results obtained from the numerical example:

1. The initial displacements of cladding panels to wind loading are erratic and considerably larger than the displacements given by the quasi-static analysis. After a short period, which is determined by the damping in the system, they become fairly consistent with the fluctuating wind load and their magnitudes are very close to the mean values.

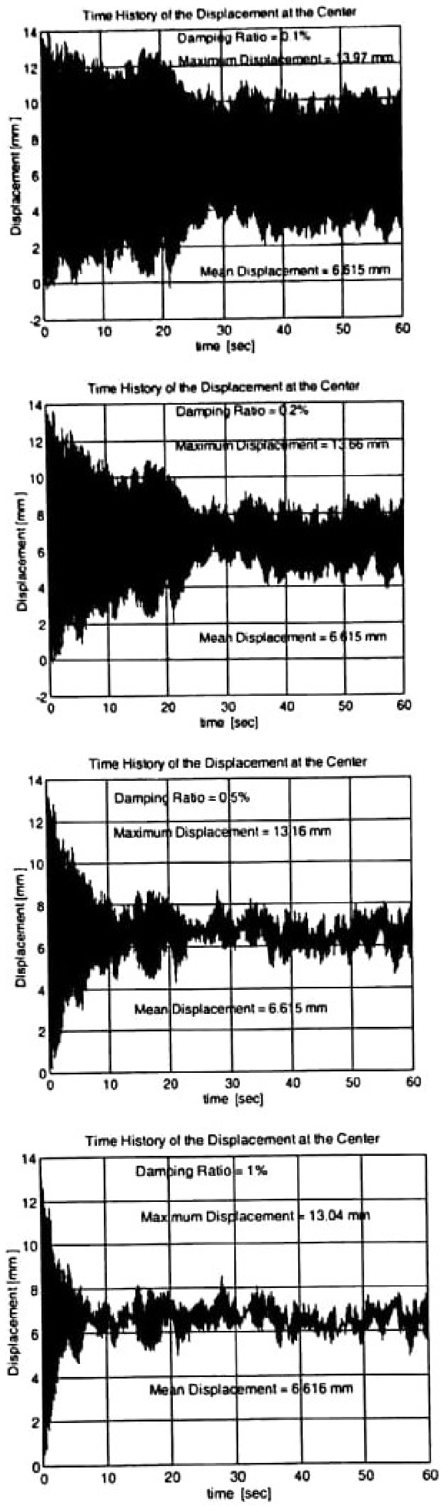

Figure 6: The Influence of the Damping on the Response of the Panel to Wind Load

2. The mean value of the dynamic response is very close to the response obtained from the quasi-static analysis.

3. The design of sensitive cladding panels such as glass and assessment of the structural integrity of existing cladding panels should be based on a dynamic analysis because the quasi-static analysis can not reproduce the initial response, which is large and erratic.

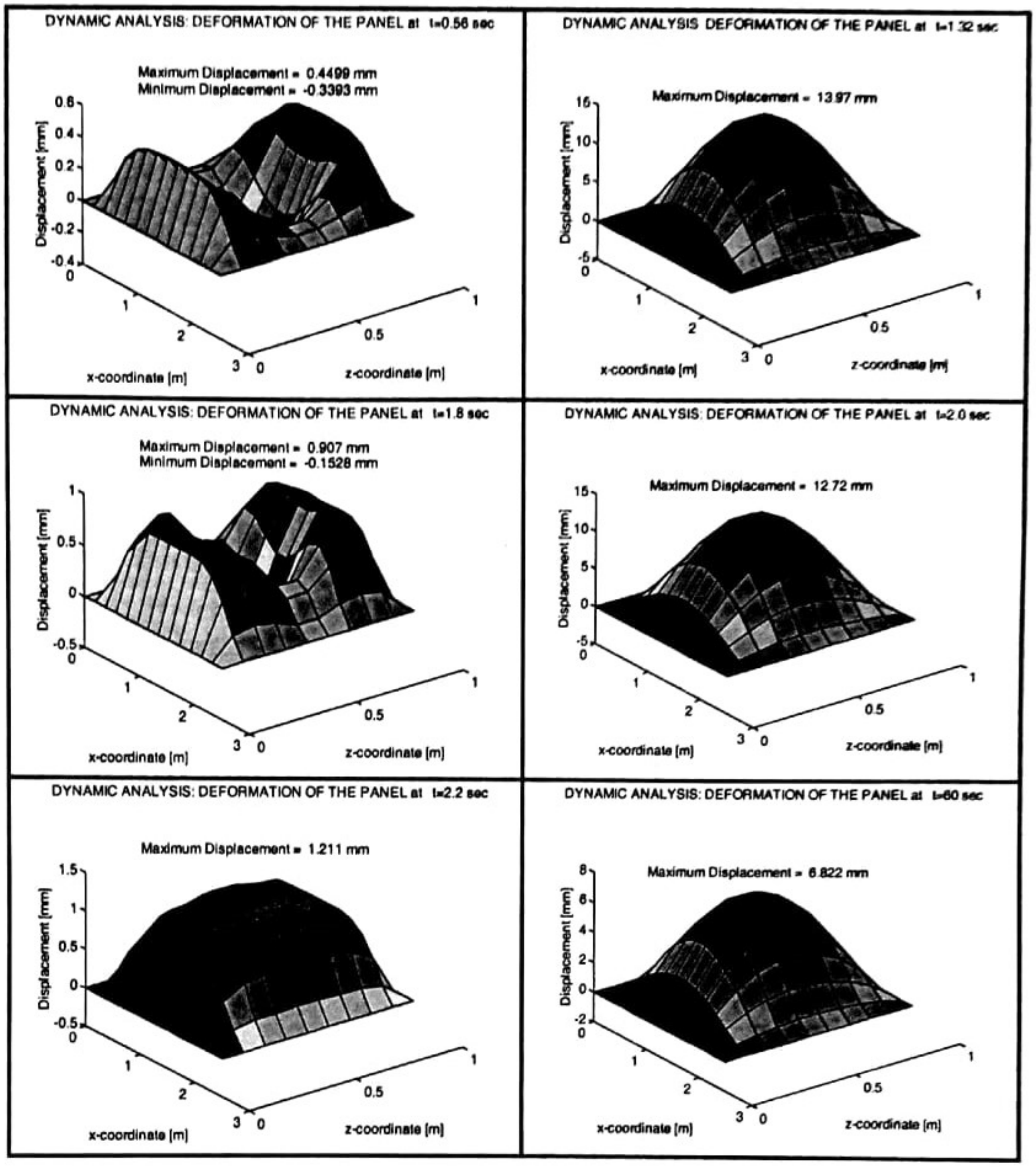

Figure7: Dynamic Response: Shape of the Panel at Various Times due to Wind Force (time is measured from the moment when the wind hit the panel)

4. Since a mathematical model is used to simulate the fluctuating wind velocity and pressure in this study, using real measured data of wind velocity and wind pressure will provide more accurate dynamic response of cladding panels to wind loading.

5. The study can be extended to cover various factors, which can lead to a non-linear behaviour such as large deflections of the panels and gaps, which may be between adjacent panels.

NOMENCLATURE

A = Area of the cladding panel in [m^2]

A_k = Amplitude of the harmonic component with the frequency η_k in [m/s]

C_D = Drag coefficient for the mean wind velocity in [-]

C'_D = Drag coefficient for the fluctuating component of the wind velocity in [-]

C_{da} = Coefficient for the aerodynamic damping force in [-]

$ce(z)$ = exposure coefficient accounting for the terrain and height above ground in [-]

$\overline{C}_p$ = pressure coefficient in [-]

c_p = pressure coefficient in [-]

F_a = Self excited forces that arise from Aero elastic phenomenon in [N]

F_D = Drag force component of the wind action in [N]

$\boldsymbol{F_D}$ = Vector of damping forces in [N]

C_M = virtual mass coefficient .[-]

f_k = the value of the frequency in the centroid of interval k in [cycles/second]

$\underline{\boldsymbol{H}}$ = an $N \times N$ transformation matrix [-]

$\boldsymbol{K}$ = Stiffness matrix of the cladding pane in [N/M]

Lu = Length scale in [m]

$\boldsymbol{M}$ = Mass matrix of the cladding panel in [kg]

$\boldsymbol{P}$ = Vector of the equivalent nodal wind loads in [N]

$R(i,j)$ = cross correlation of the wind velocities at points i and j on the panel

$r_{i,j}$ = distance between the two points

$\boldsymbol{U_t}$ = Vector of the nodal displacements at time t in [m]

$\dot{U}_t$ = Vector of the nodal velocities at time t in [m/s]

$\ddot{U}_t$ = Vector of the nodal accelerations at time t in [m/s^2]

$\bar{v}$ = mean wind velocity in [m/s]

$\bar{v}_z$ = mean wind velocity at a Height z m above the ground in [m/s]

$v(t)$ = fluctuating component of the wind velocity in [m/s]

$\underline{v}(t)$ = $[v_1(t) \quad v_2(t) \ldots v_i(t) \ldots v_N(t)]^T$ = Correlated wind velocity fluctuations in [m/s]

V_{air} = Volume of air participating in the motion of the panel in [m^3]

$v_R(\mathrm{t})$ = relative velocity in [m/s]

$\dot{v}_R(\mathrm{t})$ = relative acceleration in [m/s^2]

v_{ref} = reference wind velocity

$Y_i(t)$ = Random process representing the wind velocity in [m/s]

$\underline{Y}(t)$ = $[Y_1(t) \quad Y_2(t) \ldots Y_i(t) \ldots Y_N(t)]^T$ = The uncorrelated processes in [m/s]

α = roughness coefficient

β = interpolation parameter

δ = interpolation parameter

Δt = time step in [sec.]

Δr = Separation in [m]

φ_{ik} = random phase angles in [rad], uniformly distributed between 0 and 2π.

κ = drag coefficient referred to mean velocity at 10m in [-]

ρ = air density in [kg/m^3]

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